Generation of squeezed light in a nonlinear asymmetric directional coupler

Faisal A. A. El-Orany[*], J. Peřina[1] and M. Sebawe Abdalla[2]

[1] *Joint Laboratory of Optics of Palacký University and*

Physical Institute of Academy of Sciences of Czech Republic,

17. listopadu 50, 772 07 Olomouc, Czech Republic.

[2]*Mathematics Department, College of Science,*

King Saud University, P.O. Box 2455, Riyadh 11451, Saudi Arabia

We show that a nonlinear asymmetric directional coupler composed of a linear waveguide and a nonlinear waveguide operating by nondegenerate parametric amplification is an effective source of single-mode squeezed light. This is has been demonstrated, under certain conditions and for specific modes, for incident coherent beams in terms of the quasiprobability functions, photon-number distribution and phase distribution.

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I. INTRODUCTION

Squeezed states of light offer possibilities of improving the performance of optical devices since they can reduce fluctuations in one quadrature below the level associated with the vacuum states. Such a situation is relevant for the optical communication networks as well as for many optical devices. For instance, recently such light has been used in a power recycled interferometer [1] and in a phase-modulated signal-recycled interferometer [2] aiming to improving significantly the sensitivity of these devices. It has been shown that this light can be used to tune the resonant frequency of the cavity without actually moving the

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[*] Permanent address: Suez Canal university, Faculty of Science, Department of mathematics and computer science, Ismailia, Egypt.

signal recycling mirror or changing the bandwidth of the interferometer without substantially decreasing the sensitivity at the resonant frequency [2]. Also squeezed light has been applied in quantum information theory, e.g. in a quantum teleportation [3], cryptography [4, 5] and dense coding [6]. In this respect the security in a quantum cryptography [5] relies on the uncertainty relation for field quadrature components of these states. Further, experiments on quantum teleportation have been successfully performed by means of two-mode squeezed vacuum states [7]. A variety of methods have been proposed for the generation of squeezed states of the electromagnetic field and several of them have been realized experimentally [8, 9]. For more complete information about squeezed light the reader can consult review papers [10].

As is well known the ideal single-mode squeezed light (with possible 100% squeezing) may be generated using a degenerate parametric amplifier (DPA) with classical pump. This is the most effective system in nonlinear optics in which such strong squeezing was obtained. DPA is the two-photon process where the signal and idler modes are identical in this case. Such a property manifests [11] itself in the large scale oscillations in the photon-number distribution [12] and in the bifurcation in phase distribution [13]. In other words, the occurrence of squeezing in the quadrature variances does not need to be accompanied by oscillations in the photon-number distribution or bifurcation in the phase distribution. The best example are the binomial states [14]. On the other hand, the nondegenerate parametric amplifier (NDPA) can produce ideal squeezing in the compound modes even if the single mode squeezing does not appear [15].

In this article we suggest a compound system–nonlinear three-mode asymmetric directional coupler which is able to provide both strong single mode squeezing and compound two mode squeezing under certain conditions and for specific modes. This means that the present system can operate simultaneously as degenerate and nondegenerate amplifiers. Moreover, the single mode squeezing can be confirmed in the behaviour of both the photon-number distribution and phase distribution. In general, the nonlinear directional coupler consists of two or more parallel optical waveguides fabricated from nonlinear material. Both waveguides are placed close enough to permit flux-dependent transfer of energy between them as a consequence of evanescent waves involved in the interaction [16]. This flux transfer can be controlled by the device design and the input flux. Further, the nonlinear directional coupler has been experimentally implemented , e.g. using semiconductor/glass composite

[17], erbium doped fibers [18, 19] and a crystalline organic semiconductor [20]. Furthermore, the directional couplers are of the technological interest since they are currently a topic of major interest in integrated optics for ultra-high-speed optical signal processing and optical switching [21] as well as in the generation and transmission of nonclassical light. In the view of these reasons it is certainly useful to make concrete efforts to study the actual possibility to generate and to transfer squeezed light in this system.

It is important to mention various works investigating similar systems, e.g nonlinear asymmetric directional couplers which consist of a linear waveguide and a nonlinear waveguide operating by second harmonic generation [22–24] or of a nonlinear waveguide with the core of Kerr-like medium [25]; it has been shown that such systems can be used to reduce the necessary switching power (for the switching of a signal controlled by a strong pump) [26] as well as they are useful for constructing a band-pass power-filters or a band-reject power filter [25]. For a review of the role of quantum statistical properties in nonlinear couplers, see [27].

In the present work, we show that a nonlinear asymmetric directional coupler composed of a linear waveguide and a nonlinear waveguide operating by nondegenerate parametric amplification is an effective source of squeezed light. This will be done as follows: In section 2 we describe the model under discussion and give the solution of the equations of motion. In section 3 we demonstrate the single-mode quasidistribution functions. Sections 4 and 5 are devoted to the photon-number distribution and phase distribution, respectively. Finally we summarize the main conclusions in section 6.

II.   EQUATIONS OF MOTION

We consider a device (nonlinear asymmetric directional coupler) as outline in Fig. 1. It can be described by the Hamiltonian

$$\frac{\hat{H}}{\hbar} = [\sum_{j=1}^{3} \omega_j \hat{a}_j^\dagger \hat{a}_j] + i\lambda_1 \left\{ \hat{a}_1 \hat{a}_2 \exp[i(\omega_1 + \omega_2)t] - \text{h.c.} \right\}$$

$$+i\lambda_2 \left\{ \hat{a}_1 \hat{a}_3^\dagger \exp[i(\omega_1 - \omega_3)t] - \text{h.c.} \right\} + i\lambda_3 \left\{ \hat{a}_2 \hat{a}_3^\dagger \exp[i(\omega_2 - \omega_3)t] - \text{h.c.} \right\}, \tag{1}$$

where $\hat{a}_j(\hat{a}_j^\dagger)$, $\quad j = 1, 2, 3$ are the annihilation (creation) operators designated to the signal, idler and linear modes, respectively, λ_1 and λ_j, $\quad j = 2, 3$ are the corresponding nonlinear and linear coupling constants; ω_j are the natural frequencies of oscillations of the uncoupled

modes and h.c. is the Hermitian conjugate. The linear exchange between the two waveguides establishes through evanescent wave provided that $\omega_1 \simeq \omega_2 \simeq \omega_3$. Moreover, we treat the problem of propagation in the Hamiltonian formalism neglecting dispersion. Thus if all waves are propagating with the same velocity, time t and space z relate by the velocity of propagation v, $z = vt$. For further details of quantum description of propagation we refer the reader to [28] (and references therein). On the other hand, as we assume one-passage propagation, losses in the beams can be neglected. In general they can be described in the standard quantum way in the form of interaction of light beams with reservoirs, as for instance described in [29]. However, one can note that nonclassical properties of light beams degrade by the effect of the reservoir, less by the damping which may be by several orders weaker than the nonlinear coupling, more by the influence of non-zero mean numbers of reservoir oscillators.
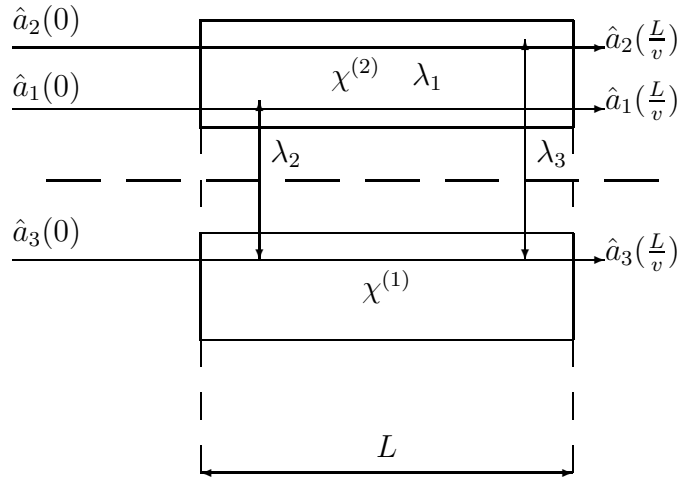

Fig.1: *Scheme of realization of interaction in (1) using a nonlinear asymmetric directional coupler which is composed of two optical waveguides fabricated from first-order ($\chi^{(1)}$) and second-order ($\chi^{(2)}$) materials, where χ designate susceptibility. Signal mode 1 and idler mode 2 propagate in the first waveguide and linear mode 3 in the second waveguide. The interaction between the signal and idler modes is established by strong pump coherent light, which is not indicated in figure, with the coupling constant λ_1. The interactions between the linear mode and the signal and idler are established linearly with the coupling constants λ_2 and λ_3, respectively. The beams are described by the photon annihilation operators as indicated; $z = vt$ is the interaction length and we assume that all beams have the same velocity v and the length of the waveguides is L. Outgoing fields are examined as single or compound*

modes by means of homodyne detection to observe squeezing of vacuum fluctuations, or by means of a set of photodetectors to measure photon correlations, photon antibunching and sub-Poissonian photon statistics in the standard ways.

The dynamics of the system is described by the Heisenberg equations of motion which, using the slowly varying forms $(\hat{a}_j = \hat{A}_j \exp(-i\omega_j t), \quad j = 1, 2, 3)$ to eliminate free oscillations so that only the slow dynamics due to the coupling between modes appear explicitly, read

$$\frac{d\hat{A}_1}{dt} = -\lambda_1 \hat{A}_2^\dagger - \lambda_2 \hat{A}_3,$$
$$\frac{d\hat{A}_2}{dt} = -\lambda_1 \hat{A}_1^\dagger - \lambda_3 \hat{A}_3,$$
$$\frac{d\hat{A}_3}{dt} = \lambda_2 \hat{A}_1 + \lambda_3 \hat{A}_2. \tag{2}$$

These are three equations with their Hermitian conjugates forming a closed system which can be solved easily by the Laplace transformation, restricting ourselves to the case $\lambda_2 = \lambda_3 = \frac{\lambda_1}{\sqrt{2}}$, to avoid the complexity in the calculations; this means that we consider stronger nonlinearity in the first waveguide compared with the linear exchange between waveguides, thereby having the solution

$$\hat{A}_1(t) = \hat{a}_1(0)f_1(t) + \hat{a}_1^\dagger(0)f_2(t) - \hat{a}_2(0)f_3(t)$$
$$-\hat{a}_2^\dagger(0)f_4(t) - \hat{a}_3(0)f_5(t) - \hat{a}_3^\dagger(0)f_6(t),$$

$$\hat{A}_2(t) = \hat{a}_2(0)g_1(t) + \hat{a}_2^\dagger(0)g_2(t) - \hat{a}_1(0)g_3(t)$$
$$-\hat{a}_1^\dagger(0)g_4(t) - \hat{a}_3(0)g_5(t) - \hat{a}_3^\dagger(0)g_6(t),$$

$$\hat{A}_3(t) = \hat{a}_3(0)h_1(t) + \hat{a}_3^\dagger(0)h_2(t) + \hat{a}_2(0)h_3(t)$$
$$+\hat{a}_2^\dagger(0)h_4(t) + \hat{a}_1(0)h_5(t) + \hat{a}_1^\dagger(0)h_6(t), \tag{3}$$

where the time-dependent coefficients, i.e. $f_j(t), g_j(t), h_j(t)$, including all information about the system are

$$f_{1,3}(t) = \frac{1}{2}\left[\cosh(\lambda_1 t) \pm \cosh(\frac{\lambda_1 t}{2})\cos(\bar{k}t) \pm \frac{1}{\sqrt{3}}\sinh(\frac{\lambda_1 t}{2})\sin(\bar{k}t)\right],$$
$$f_{2,4}(t) = \frac{1}{2}\left[\sinh(\lambda_1 t) \mp \sinh(\frac{\lambda_1}{2}t)\cos(\bar{k}t) \mp \frac{1}{\sqrt{3}}\cosh(\frac{\lambda_1 t}{2})\sin(\bar{k}t)\right],$$
$$f_5(t) = \sqrt{\frac{2}{3}}\cosh(\frac{\lambda_1 t}{2})\sin(\bar{k}t), \quad f_6(t) = -\sqrt{\frac{2}{3}}\sinh(\frac{\lambda_1 t}{2})\sin(\bar{k}t), \tag{4}$$

$$h_1(t) = \cosh(\frac{\lambda_1 t}{2})\cos(\bar{k}t) - \frac{1}{\sqrt{3}}\sinh(\frac{\lambda_1 t}{2})\sin(\bar{k}t),$$

$$h_2(t) = -\sinh(\frac{\lambda_1 t}{2})\cos(\bar{k}t) + \frac{1}{\sqrt{3}}\cosh(\frac{\lambda_1 t}{2})\sin(\bar{k}t),$$

$$h_{3,5}(t) = \sqrt{\frac{2}{3}}\cosh(\frac{\lambda_1 t}{2})\sin(\bar{k}t), \quad h_{4,6}(t) = -\sqrt{\frac{2}{3}}\sinh(\frac{\lambda_1 t}{2})\sin(\bar{k}t). \tag{5}$$

where $\bar{k} = \frac{\sqrt{3}}{2}\lambda_1$ and the expressions for $g_j(t)$ are the same as those of $f_j(t)$.

In fact, the nature of the solution can show how the coupler does work. To be more specific, the time-dependent coefficients contain both trigonometric and hyperbolic functions. Consequently, the propagating beams inside the coupler can be amplified as well as switched between waveguides, i.e. between modes, in the course of time.

On the basis of the well known commutation rules for boson operators, the following relations can be proved for the time-dependent coefficients

$$f_1^2(t) - f_2^2(t) + f_3^2(t) - f_4^2(t) + f_5^2(t) - f_6^2(t) = 1,$$

$$f_1(t)g_4(t) - f_2(t)g_3(t) + f_3(t)g_2(t) - f_4(t)g_1(t) - f_5(t)g_6(t) + f_6(t)g_5(t) = 0,$$

$$f_1(t)g_3(t) - f_2(t)g_4(t) + f_3(t)g_1(t) - f_4(t)g_2(t) - f_5(t)g_5(t) + f_6(t)g_6(t) = 0. \tag{6}$$

The remaining relations can be obtained from (6) by means of the following transformations

$$\left(f_1(t), f_2(t), f_3(t), f_4(t), f_5(t), f_6(t) \right)$$

$$\longleftrightarrow \left(-g_3(t), -g_4(t), -g_1(t), -g_2(t), g_5(t), g_6(t) \right)$$

$$\longleftrightarrow \left(h_5(t), h_6(t), -h_3(t), -h_4(t), -h_1(t), -h_2(t) \right). \tag{7}$$

Based on the results of the present section, we can study the quantum properties of the evolution of different modes in the model when they are initially prepared in coherent states. This will be demonstrated in the following sections.

III. QUASIPROBABILITY FUNCTIONS

Evaluation of various time-dependent mode observable is most conveniently achieved with the aid of corresponding time-dependent characteristic functions, their normal, antinormal and symmetric forms, and the Fourier transforms of these characteristic functions (quasiprobability functions). All of these are related to the density matrix which provides a complete statistical description of the system. There are three types of quasiprobability functions: Wigner W-, Glauber P-, and Husimi Q-functions. These functions could be used also as crucial to describe the nonclassical effects of the system, e.g. one can employ the negative values of W-function, stretching of Q-function and high singularities in P-function. Furthermore, these functions are now accessible from measurements [30].

In the following we consider phase space distributions for the single-mode when all modes are initially prepared in coherent states before entering the coupler.

We start from knowledge of the single-mode s-parameterized characteristic function which is suitable to describe the quantum statistics of light and for the jth mode it is defined by

$$C_j(\zeta, t, s) = \text{Tr}\left\{\hat{\rho}(0)\exp\left[\zeta\hat{A}_j^\dagger(t) - \zeta^*\hat{A}_j(t) + \frac{s}{2}|\zeta|^2\right]\right\}, \tag{8}$$

where $\hat{\rho}(0)$ is the initial density matrix for the system under consideration, j takes on the values $1, 2, 3$ corresponding to considered mode and s takes on values $1, 0$ and -1 corresponding to normally, symmetrically and antinormally ordered characteristic functions, respectively. Assume that the modes are initially uncorrelated and are described by the density operator

$$\hat{\rho}(0) = \prod_{j=1}^{3}|\alpha\rangle_{jj}\langle\alpha|, \tag{9}$$

where $|\alpha\rangle_j$ are coherent states. So that equations (8) and (9) lead to the single-mode s-parameterized characteristic function which for the signal mode takes the form

$$C_1(\zeta, s, t) = \exp\left\{\frac{1}{2}|\zeta|^2[s - |\eta_1(t)|] + |\eta_2(t)|(\zeta^2 + \zeta^{*2} + [\bar{\alpha}_1^*(t)\zeta - \bar{\alpha}_1(t)\zeta^*]\right\}, \tag{10}$$

where $\bar{\alpha}_1(t)$ is the mean value of the operator $\hat{A}_1(t)$ in the coherent state and $\eta_j(t)$, $\quad j = 1, 2$, are given by

$$\eta_1(t) = f_1^2(t) + f_2^2(t) + f_3^2(t) + f_4^2(t) + f_5^2(t) + f_6^2(t),$$
$$\eta_2(t) = f_1(t)f_2(t) + f_3(t)f_4(t) + f_5(t)f_6(t). \tag{11}$$

The single-mode s-parameterized quasiprobablity function is defined as

$$W_j(\beta, t, s) = \frac{1}{\pi^2}\int C_j(\zeta, t, s)\exp(\beta\zeta^* - \beta^*\zeta)d^2\zeta, \tag{12}$$

where $C_j(\zeta, t, s)$ is the s-parameterized single-mode characteristic function and when $s = 1, 0, -1$ relation (12) gives P-, W- and Q-functions, respectively.

Now the s-parameterized single-mode quasiprobability functions of the signal mode, which can be derived from (10) and (12), are

$$W_1(\beta, s, t) = \frac{2}{\pi\sqrt{[A_+(t) - s][A_-(t) - s]}}$$
$$\times \exp\left\{\frac{[l_1(t) - l_1^*(t)]^2}{2[A_-(t) - s]} - \frac{[l_1(t) + l_1^*(t)]^2}{2[A_+(t) - s]}\right\}, \tag{13}$$

where $l_1(t) = \bar{\alpha}_1(t) - \beta$, $\bar{\alpha}_1(t)$ has the same meaning as before and

$$A_\pm(t) = [f_1(t) \pm f_2(t)]^2 + [f_3(t) \pm f_4(t)]^2 + [f_5(t) \pm f_6(t)]^2; \tag{14}$$

the other expressions for the idler and linear modes can be obtained from (13) and (14) by making use of the transformations (7); we except here the case $s = 1$, which will be discussed at the end of this section. Now let us start our investigation by demonstrating the behaviour of the W-function, i.e. setting $s = 0$ in (13). In the language of mechanical

analogy of a harmonic oscillator with dynamical conjugate variables x and y, i.e $\beta = x + iy$, the W-function can be written in the form

$$W_1(x, y, t) = \frac{1}{2\pi\sqrt{\langle(\triangle\hat{X}(t))^2\rangle\langle(\triangle\hat{Y}(t))^2\rangle}} \exp\left\{ -\frac{[x - \bar{\alpha}_x(t)]^2}{2\langle(\triangle\hat{X}(t))^2\rangle} - \frac{[y - \bar{\alpha}_y(t)]^2}{2\langle(\triangle\hat{Y}(t))^2\rangle} \right\}, \quad (15)$$

where

$$\langle(\triangle\hat{X}(t))^2\rangle = \frac{A_+(t)}{4}, \qquad \langle(\triangle\hat{Y}(t))^2\rangle = \frac{A_-(t)}{4}, \quad (16)$$

where $\bar{\alpha}_1(t) = \bar{\alpha}_x(t) + i\bar{\alpha}_y(t)$; $\langle(\triangle\hat{X}(t))^2\rangle$ and $\langle(\triangle\hat{Y}(t))^2\rangle$ are x- and y-quadrature variances, respectively. As known these quadratures relate to the conjugate electric and magnetic field operators of the electromagnetic field and may be used as a measure of squeezing phenomenon. More illustratively, the two quadrature operators of the jth single-mode are defined through $\hat{X}(t) = \frac{1}{2}[\hat{A}_j(t) + \hat{A}_j^\dagger(t)]$, $\hat{Y}(t) = \frac{1}{2i}[\hat{A}_j(t) - \hat{A}_j^\dagger(t)]$, where $[\hat{X}(t), \hat{Y}(t)] = \frac{i}{2}$ and then the uncertainty-relation reads $\langle(\triangle\hat{X}(t))^2\rangle\langle(\triangle\hat{Y}(t))^2\rangle \geq \frac{1}{16}$ where, e.g. $\langle(\triangle\hat{X}(t))^2\rangle = \langle(\hat{X}(t))^2\rangle - \langle\hat{X}(t)\rangle^2$. Therefore, we can say that the jth mode is squeezed if $4\langle(\triangle\hat{X}(t))^2\rangle - 1 < 0$ or $4\langle(\triangle\hat{Y}(t))^2\rangle - 1 < 0$. Now from (14) and (16) together with (5) one can easily prove that the quadrature variances of the linear mode (single-mode squeezing) are

$$\langle(\triangle\hat{X}(t))^2\rangle = \frac{1}{4}\left\{ \frac{4}{3}\sin^2(\bar{k}t) + [\cos(\bar{k}t) + \frac{1}{\sqrt{3}}\sin(\bar{k}t)]^2 \right\} \exp(-\lambda_1 t), \quad (17)$$

$$\langle(\triangle\hat{Y}(t))^2\rangle = \frac{1}{4}\left\{ \frac{4}{3}\sin^2(\bar{k}t) + [\cos(\bar{k}t) - \frac{1}{\sqrt{3}}\sin(\bar{k}t)]^2 \right\} \exp(\lambda_1 t), \quad (18)$$

where $\bar{k}$ has the same meaning as before. Thus squeezing can be achieved in the linear waveguide in the X-quadrature. Moreover, it is clear that squeezing values become more pronounced for a large interaction time, i.e. for long length of the coupler L. This behaviour shows that changing the power of the linear interaction it is possible to transfer nonclassical properties, which are generated in the nonlinear waveguide, to the linear signal mode. For completeness, the uncertainty relation reads

$$\langle(\triangle\hat{X}(t))^2\rangle\langle(\triangle\hat{Y}(t))^2\rangle = \frac{1}{16}[1 + \frac{16}{9}\sin^4(\bar{k}t)]. \quad (19)$$

This formula reveals that the minimum-uncertainty relation holds only when $t = t_s = \frac{2m\pi}{\sqrt{3}\lambda_1}$ where m is a positive integer and we call t_s a squeezed time. In this case the device provides the squeezed coherent light in the linear mode with squeeze parameter $r = \frac{2m\pi}{\sqrt{3}}$. It is important to mention that at $t = t_s$ the two waveguides become completely independent (cf. equations (3)-(5)) and the signal and idler modes can display perfect two-mode squeezing, which one can easily check with the help of two-mode quadrature variances. This can be

explained as follows: A portion of the energy always remains within the guide into which the field was initially injected. This energy grows in the course of time until $t = t_s$, when the two guides are completely independent and hence the modes are trapped in their own guides. So that the self-interaction of the linear mode can attribute the well known single-mode squeezed light in the linear waveguide having its origin in the nonlinear waveguide in which the nonlinearity produces two-mode squeezing. For later times $t \neq t_s$, the device generates single-mode squeezed light only in the linear waveguide where the corresponding W-function may be broader than that of the squeezed coherent states. This situation is close to that of a two-photon absorber (a two-photon absorption by a reservoir of two-level atoms from a single mode of the electromagnetic field [31]) where a squeezed state, which is not a minimum-uncertainty state, has been generated [32]. The origin of squeezing of initially unsqueezed light interacting with two-photon absorbers is that the squeezing is generated by simple quantum superposition of states of light [31].

We proceed in our discussion by focusing our attention on the P-function. It is known that the correspondence between quantum and classical theories can be established with the use of this function. However, P-representation does not have all the properties of a classical distribution function, especially for quantum fields. To be more specific, light fields for which the P-representation is not well-behaved distribution (in most processes involving an interaction at least for some values of interaction time, including the process under consideration) can exhibit nonclassical features. We have shown earlier that this model is able to provide squeezed light and this should be reflected in the behaviour of the P-representation, i.e. setting $s = 1$ in (13). The significant example for this situation is the behaviour of the linear mode. For this case, we can show that the single-mode quadrature squeezing is established provided that $A_+(t) - 1 < 0$ and $A_-(t) - 1 > 0$. It is evident that the P-function is not well-behaved function in this case and this is the indication of the nonclassical fields. Indeed, if we look at the model under consideration as a competition of parametric processes, we can find that this result is in marked contrast to the most results occurred in the literature for three interacting modes [33, 34], where the delta function of the initial P-representation of coherent light for a single mode becomes well-behaved distribution during the interaction.

In the following sections we use the phase space distribution functions to study the photon-number distribution and phase distribution for the system under discussion.

IV.   PHOTON-NUMBER DISTRIBUTION

Photon-number distribution, i.e. the probability of finding n_1 photons in the signal mode at time t, can be obtained in the photodetection process, and is determined by means of the relation [33]

$$P(n_1, t) = \langle n_1|\hat{\rho}_1(t)|n_1\rangle = \int \frac{|\beta|^{2n_1}}{n_1!} W(\beta, s = 1, t) \exp(-|\beta|^2)d^2\beta; \tag{20}$$

substituting (13) into (20) and carrying out the integration, we get the photon-number distribution of the signal mode as

$$P(n_1, t) = \frac{2}{\sqrt{[A_+(t) + 1][A_-(t) + 1]}} \exp\left\{\frac{[\bar{\alpha}_1(t) - \bar{\alpha}_1^*(t)]^2}{2[A_-(t) + 1]} - \frac{[\bar{\alpha}_1(t) + \bar{\alpha}_1^*(t)]^2}{2[A_+(t) + 1]}\right\}$$

$$\times \sum_{r=0}^{n_1} \left[\frac{A_+(t) - 1}{A_+(t) + 1}\right]^r \left[\frac{A_-(t) - 1}{A_-(t) + 1}\right]^{n_1 - r}$$

$$\times \mathrm{L}_r^{-\frac{1}{2}}\left[-\frac{(\bar{\alpha}_1(t) + \bar{\alpha}_1^*(t))^2}{A_+^2(t) - 1}\right] \mathrm{L}_{n_1 - r}^{-\frac{1}{2}}\left[\frac{(\bar{\alpha}_1(t) - \bar{\alpha}_1^*(t))^2}{A_-^2(t) - 1}\right], \tag{21a}$$

where $L_n^k(x)$ is the associated Laguerre polynomials defined as

$$L_n^k(x) = \sum_{m=0}^{n} \frac{\Gamma(n + k + 1)(-x)^m}{(n - m)!\Gamma(m + k + 1)m!}, \tag{21b}$$

while Γ is the Gamma function.

It is interesting to compare this distribution with the corresponding Poisson distribution

$$P(n_1, t) = \frac{\langle \hat{n}_1(t)\rangle^{n_1}}{n_1!} \exp(-\langle \hat{n}_1(t)\rangle), \tag{22}$$

which corresponds to fully coherent field with the same mean photon number $\langle \hat{n}_1(t)\rangle$. The expressions for the idler and linear modes should be obtained from (21a) using the transformations (7).

In Fig. 2, we have plotted $P(n_1, t = 1.5)$ against n_1 for the signal mode (solid curve) for $\alpha_j = 3\exp(i\frac{\pi}{3})$ and $\lambda_1 = 0.3$. For the sake of comparison, the corresponding photon distribution for coherent field, (22), is shown by dashed curve. For all numerical calculations $\sum_{n_1=0}^{\infty} P(n_1, t) = 1$ with good accuracy. We noted for the signal mode that the photon-number distribution exhibits always one-peak structure. From this figure we see that the behaviour is rather super-Poissonian as a result of quantum fluctuations because the solid curve is always broader than the corresponding dashed curve. So that, in general the Poissonian light evolves in the nonlinear waveguide as super-Poissonian light. On the other hand, as we have realized earlier from the behaviour of W-function that squeezed states can be generated. However, these states are exhibiting oscillating photon-number distribution for certain values of squeeze parameter. These oscillations are purely quantum effect
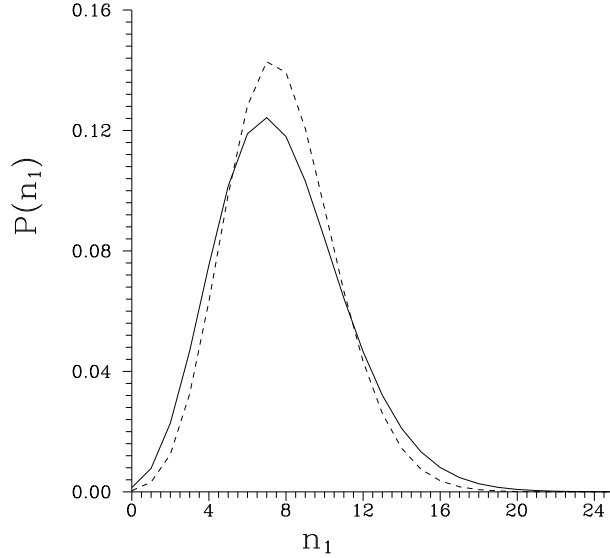
FIG. 2: Photon-number distribution $P(n_1)$ for the signal mode for $\alpha_j = 3\exp(i\frac{\pi}{3}), j = 1,2,3, \quad t = 1.5$ and $\lambda_1 = 0.3$. The dashed curve is the Poisson photon-number distribution.

without classical analogue and they can be interpreted as interference in phase space [12] or in the framework of the generalized superposition of coherent fields and quantum noise [35]. Indeed, these oscillations can be recognized for the linear mode where this mode can display squeezed light (see Fig. 3, for shown values of parameters). From this figure we see the macroscopic oscillations of the photon-number distribution for squeezed light. Further, comparison of Figs. 3a and 3b shows that the nonclassical oscillations of $P(n_3)$ are faster and more pronounced when t increases. Moreover, we noted that these oscillations appear only when the intensities of the input light are weak otherwise the single-peak structure is dominant. The broader oscillation range means that $\langle (\triangle \hat{n}_3(t))^2 \rangle > \langle \hat{n}_3(t) \rangle$ and thus there are always super-Poissonian statistics for the linear mode. It is worthwhile mentioning that the case of Fig. 3a corresponds to squeezed light with squeeze parameter $r \simeq 3.6$.

In fact, the behaviour of the photon-number distribution in this device is slightly different from that of the nonlinear asymmetric coupler with strong classical stimulating light in the second harmonic mode [23] where oscillatory behaviour as well as sub-Poissonian statistics for specific modes in both linear and nonlinear waveguide are exhibited.
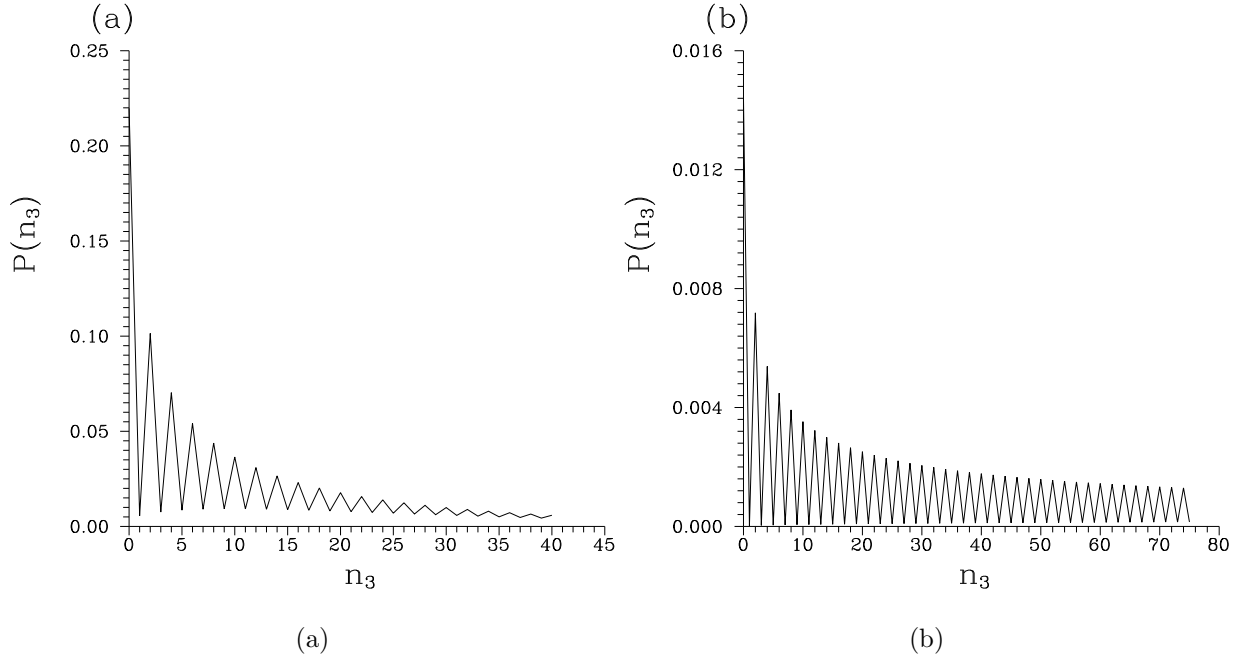
(a)

(b)

FIG. 3: Photon-number distribution $P(n_3)$ for the linear mode when $\alpha_j = 0.5\exp(i\frac{\pi}{3}), j = 1, 2, 3; \lambda_1 = 1$ and a) $t = t_s$; b) $t = 5$.

V. PHASE DISTRIBUTION

Nonlinear optical phenomena are sources of optical fields, the statistical properties of which are changed as a result of the nonlinear interaction. Quantum phase properties are among those statistical properties which undergo nonlinear changes. The interest in the phase properties has been motivated by experimental realization of optical homodyne tomography [36] allowing quantum phase from the measured field density matrix.

We make use of the single-mode Q-function to investigate the phase distribution for the system under discussion by integrating this function over the radial variable [13]

$$P(\Theta, t) = \int_0^\infty W(\beta, s = -1, t)|\beta|d|\beta|. \tag{23}$$

It is worth mentioning that the authors of [37, 38] have defined a Hermitian relative phase operator, which is in fact a polar decomposition of the Stokes operators for the two-mode field. In order to measure the single-mode phase distribution in this approach we have to prepare one of the modes in a very intense state of well defined phase and then the phase information of the other mode can be estimated [37]. Recently, this approach has been experimentally demonstrated [39]. Actually, this approach gives different behaviour than that proposed from quasiprobability distribution functions for weak fields because of variety

of possible definitions of quantum phase [13]. However, they are in a good agreement for strong fields [40].

We proceed, equations (13) and (23) lead to

$$P(\Theta, t) = \frac{1}{\pi C(t)\sqrt{A_+(t)A_-(t)}} \exp\left\{ \frac{[\bar{\alpha}_1(t) - \bar{\alpha}_1^*(t)]^2}{2[A_-(t) + 1]} - \frac{[\bar{\alpha}_1(t) + \bar{\alpha}_1^*(t)]^2}{2[A_+(t) + 1]} \right\}$$

$$\times \left\{ 1 + \frac{B(t)}{2}\sqrt{\frac{\pi}{C(t)}} \exp\left(\frac{B(t)^2}{4C(t)}\right) \left[1 + \mathrm{erf}(\frac{B(t)}{2\sqrt{C(t)}}) \right] \right\}, \tag{24}$$

where

$$C(t) = \frac{2}{A_-(t)A_+(t)}[A_+(t)\sin^2\Theta + A_-(t)\cos^2\Theta],$$

$$B(t) = \frac{2}{A_-(t)A_+(t)}\left\{ A_-(t)[\bar{\alpha}_1(t) + \bar{\alpha}_1^*(t)]\sin\Theta - iA_+(t)[\bar{\alpha}_1(t) - \bar{\alpha}_1^*(t)]\cos\Theta \right\}, \tag{25}$$

and we have used the Gauss error function, which is defined by the well-known formula

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-y^2)dy, \tag{26}$$

taking into account that $\beta = |\beta|\exp(i\Theta)$.

It is clear that (24) is 2π-periodic function and the behaviour of this distribution in this device can be understood on the basis of the competition between the two-peak structure for the vacuum states and the single-peak structure for coherent states provided that the coherent amplitudes are real. Such behaviour is demonstrated in Figs. 4a,b for the signal mode and in Figs. 5a,b for the linear mode, where the evolution of the phase distribution (24) has been depicted against the phase Θ and the time t.

In figures a and b, $\alpha_j = 0$ and 1, $j = 1, 2, 3$, respectively, for the shown value of coupling constant λ_1. In Fig. 4a, one can see the phase evolution for the vacuum states in the signal mode showing two peak-structure with one central peak at $\Theta = 0$ and two wings as $\Theta \to \pm\pi$. In fact, this behaviour has been seen for some of superposition states, e.g. for Yurke-Stoler states [41] and odd-binomial states [42]. In Fig. 4b, one can observe that the initial peak for coherent state is attenuated in the course of time while the two peaks for the vacuum states are created and amplified. The well-known behaviour for the phase distribution of squeezed states may be established when we focus our attention on the behaviour of the linear mode, as expected (see Figs. 5a and b).

From figure 5a one can see that the two-peak structure is typical for squeezed vacuum states, i.e. we have two-peak structure for $\Theta = \pm\frac{\pi}{2}$ [13]. Further, the heights of these peaks are $\frac{1}{2\pi}\sqrt{\frac{A_-(t)}{A_+(t)}}$ at any time t $(t > 0)$. However, Fig. 5b displays the well-known bifurcation shape for the phase distribution of squeezed field, i.e. the distribution curve
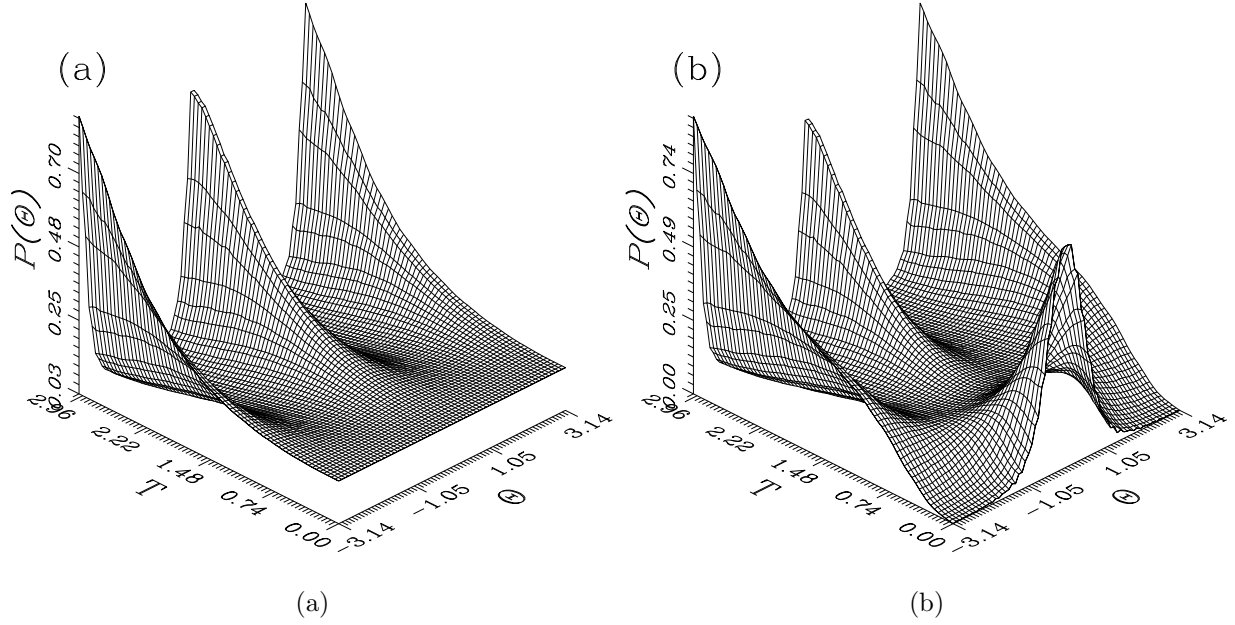
FIG. 4: Phase distribution $P(\Theta, t)$ for the signal mode against Θ and t for $\lambda_1 = 0.6$ and a) $\alpha_j = 0$; b) $\alpha_j = 1$, $j = 1, 2, 3$.

undergoes a transition from single- to a double-peaked form with increasing time. Indeed this figure is quite similar to that for squeezed coherent states [43], and the distributions differ in the behaviour of the initial peak, which is here amplified for a while before splitting into two peaks. This is connected with the significant influence of the development by nonlinear effects and power transfers between waveguides. It is worthwhile mentioning that a similar behaviour has been obtained for a contradirectional nonlinear asymmetric coupler with strong stimulated coherent field in the second harmonic waveguide [24].

Finally, we would like to conclude this section by discussing the origin of such behaviour of the phase distribution. This can be easily understood by analysing the function $P(\Theta, t)$ when $\alpha_j = 0$, $j = 1, 2, 3$. In this case the formula (24) reduces to

$$P(\Theta, t) = \frac{1}{2\pi} \frac{\sqrt{A_+(t)A_-(t)}}{A_+(t)\sin^2\Theta + A_-(t)\cos^2\Theta}. \tag{27}$$

This distribution function is double- or three-peak structure according to the relation between $A_+(t)$ and $A_-(t)$. Let us restrict our discussion to the one of the modes which are propagating in the nonlinear waveguide, say, to the signal mode. For this mode, it can easily be shown that $A_+(t) > A_-(t)$ and consequently the formula (27) exhibits three-peak structure with peaks for $\Theta = 0, \pm\pi$. The heights of these peaks at any time $t \neq 0$ are $\frac{1}{2\pi}\sqrt{\frac{A_+(t)}{A_-(t)}}$. For non-zero displacements α_j, an additional factor of the form for coherent state, but with
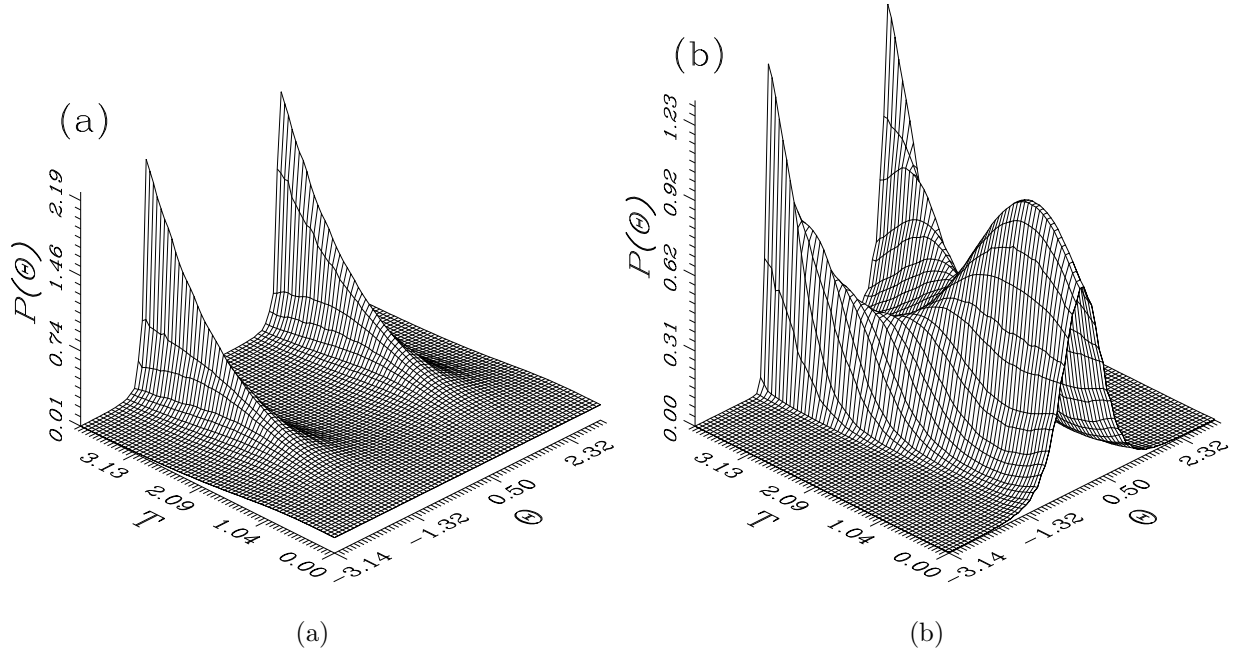
(a)

(b)

FIG. 5: Phase distribution $P(\Theta, t)$ for the linear mode against Θ and t for the same situation as in Fig. 4.

a coherent amplitude $\bar{\alpha}_1(t)$ which is the expectation value of the signal mode operator in coherent state, appears in the distribution. This factor is responsible for a peak at $\Theta = 0$ related to coherent component, which competes with the three-peak structure of vacuum to provide the previous behaviour of the distribution. Similar argument can be adopted to explain the behaviour of the linear mode, however, it should be borne in mind that in this case $A_+(t) < A_-(t)$, as we have shown earlier for squeezing phenomenon, and hence the two-peak structure for the input vacuum case is dominant at $\Theta = \pm\frac{\pi}{2}$.

VI.   CONCLUSION

In this article we have investigated quantum statistical properties of squeezed light generated in a nonlinear asymmetric directional coupler composed of a linear waveguide and a nonlinear waveguide operating by nondegenerate parametric amplification. The main difference between the present system and the well documented optical parametric oscillator (OPO) is the operating mechanism of these devices. More illustratively, OPO is using a quadratic nonlinear medium [9] in a resonator and it is a practical device to generate squeezing; the system discussed here is richer by construction and by their properties be-

cause it includes two propagating media (i.e. waveguides), where one is linear and the other is nonlinear and they exchange energy linearly via evanescent waves. The system considered here consists of a mixture of nonlinear processes including parametric generation and amplification (interaction coefficient λ_1) and frequency conversion (interaction coefficients λ_2 and λ_3). This can lead to a single mode generation of nonclassical light.

Using the Heisenberg approach we have examined quantum statistics of interacting modes, including the linear mode, propagating in the linear waveguide, in terms of the quasiprobability functions, photon-number distribution and phase distribution when the modes are initially in coherent states. From the behaviour of the Glauber P- and Wigner W-functions in the linear mode it follows that the former is not well-behaved function when $t > 0$, while, the latter displays the well-known behaviour for squeezed light in which one of the quadratures is amplified and the other is attenuated. In the photon-number distribution, the large scale macroscopic oscillations related to squeezed light are established. The phase distribution displays the bifurcation typical for squeezed field. This behaviour demonstrates that changing the power of the linear interaction, it is possible to transfer nonclassical properties, which are generated in the nonlinear waveguide, to the linear signal mode and to control them. Further, we have shown, in general, that the system is more suitable for generation of squeezed light rather than sub-Poissonian light from initial coherent light.

Acknowledgments

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